THE CUTOFF BENEFIT-COST RATIO SHOULD EXCEED ONE

EDWARD M. MILLER
University of New Orleans

ABSTRACT

The engineering economics textbook rule of accepting all projects with a benefit-cost ratio in excess of one worth fails to consider the bias introduced from there being typically more poor projects than good projects. Bayesian statistics provides an analytical solution to the problem of combining prior information with an engineer's estimates. With all distributions normal, the posterior mean (to be used for decision making) becomes a weighted average of the prior mean, and the mean derived from the engineer's estimates. A simple formula provides the optimal cut-off benefit-cost ratio. This value typically exceeds the textbook's unity cut-off benefit-cost ratio.

INTRODUCTION

Engineering economics textbooks (Eschenbach, 1995, Newman and Lavelle, 1998, Park, 1996, Park and Sharp-Bette, 1990, Riggs, Bedworth, and Randhawa, 1996, Steiner, 1996) give a simple rule for deciding whether to accept a single project, not a member of a set of mutually exclusive projects. The project should be accepted if, and only if, the ratio of the present worth of benefits to the present worth of costs exceeds unity. Yet, it has been shown (Miller 1978, 2000) that where there is a non-uniform prior and errors in estimates, the traditional criteria of accepting a project when its estimated net present worth is positive, gives wrong answers even when the calculations and data are unbiased. Incidentally, the 1978 paper pointing out the effect made the list of the most cited papers from the journal Financial Management (Borokhovich, Bricker, Zivney, & Sundaram, 1995).

One solution for the problem of what has been called "uncertainty induced bias" (Miller 1978) is to construct explicit decision trees and then to use Bayes Theorem. The minimum acceptable present worth estimate is then the present worth estimate that best equates the true present worth to the cost. However, the above procedure is relatively complex and time consuming, requiring the calculation of a separate minimum acceptable estimated benefit for each project. Only large projects can justify this much analytic effort.

It would be very useful to have a cut-off benefit-cost ratio suitable for whole classes of projects. A rule expressed as a benefit-cost ratio would work with projects of different cost. Fortunately, a simple rule can be derived from a standard Bayesian formula for combining prior and sample information. While there has been very little use made of Bayesian analysis in capital budgeting, Bayesian methods provide tools for optimally combining prior information with current information. In capital budgeting, there is typically prior knowledge. Unfortunately, the standard textbook methods fail to make use of this prior knowledge.

PRIOR KNOWLEDGE IN ENGINEERING ECONOMICS

Where does this prior knowledge come from? In some cases nature is such that there are more poor projects than good ones. It is well known that the typical research and development project fails. There are more plausible-sounding projects than there are economically viable projects. Even in the case of oil deposits, as I pointed out long ago (Miller 1969), the firm with the most optimistic geologist wins the lease. This "winner's curse" effect is prior information.

Typically, what one firm regards as a good project will also appear to be a good project to a competitor. The pool of projects nature provides is quickly depleted of good projects while the poor ones stay around to be considered again and again.

In addition, a second factor comes into play. Frequently, competition sets the price of a scarce resource necessary for the project. The economic theory of rent implies that such inputs will have been bid up in price, leaving only a normal return to investments using these resources.

These pieces of standard economic theory information, when translated into engineering economics terms, imply that the projects with benefits exceeding costs will be rare, while there will be many projects with benefits less than costs.

In Bayesian terms, the above economic knowledge constitutes prior information that should be taken into account in decision-making. In principle, Bayesian decision-making provides a way to do so.

THE SPECIAL CASE OF THE NORMAL DISTRIBUTION

For a few distributions analytic solutions are known. One of these is the very important case where both the prior and posterior distributions are normal.

The distribution of errors can be plausibly modeled as a normal distribution. The true (prior) present worth distribution may also be normal or at least approximated that way. A plot of this distribution may be a downward sloping curve, but this might be treated as the right hand part of a normal distribution.

In these circumstances, the posterior distribution will be normal. It is the posterior distribution that is needed for making engineering economics decisions.

Fortunately, an analytic solution for deriving the posterior distribution exists for normal distributions (See Dyckman, Smidt, & McAdams [1968, p. 486]). This can be used to derive a minimum acceptable benefit-cost ratio.

Let M_p = the mean present worth of the candidate projects (the prior mean)

M_e = the mean of the distribution of the present worth of the candidate project's benefits given the data about them (excluding prior information).

M_t = the expected worth for the present worth of the candidate projects given the estimates (the posterior mean)

s_p = the standard deviation of the present worths for the population of projects (the prior estimate)

s_e = the standard deviation of the estimated present worths

s_t = the standard deviation of the present worths of the candidate project's benefits given the estimate

With this notation,

$$M_t = ((M_p / s_p^2) + M_e / s_e^2)/(1/ s_p^2 + 1/ s_e^2) \qquad (1)$$

$$M_t = ((M_p / s_e^2/(s_e^2+s_p^2) + M_e s_p^2/(s_e^2+s_p^2) \qquad (2)$$

And,

$$1/ s_t^2 = 1/ s_p^2 + 1/ s_e^2 \qquad (3)$$

The mean and the standard deviation of the projects' present worths given the estimated present worths and the prior distribution are shown above. Eq. (1) gives the posterior distribution mean. It is a weighted average of the population mean for the true worths (the prior distribution) and the project's estimated present worth. The weights are:

$$\text{Weight for the engineer's evaluation} = W_e = s_e^2/(s_e^2 + s_p^2), \qquad (4)$$

and,

$$\text{Weight for the prior opinion} = W_p = s_p^2/(s_e^2 + s_p^2) \qquad (5)$$

The mean of the posterior distribution is the expected present worth for the project.

In plain English, once an initial estimate has been obtained, the best present worth estimate will depend on the average present worth of the candidate projects. In general, the effect is to move the estimate for each project towards the mean for all projects.

By definition, the best projects will usually have evaluations above the mean of all the projects considered. It follows that the best projects will typically have their returns adjusted downwards. When trying to select the best projects, each project's estimate should be lowered by an amount that depends on the mean and standard deviation for all projects, and the estimate's precision (the technical term for the reciprocal of the variance).

IMPLICATIONS FOR BENEFIT-COST RATIOS

Once the decision-maker has estimated the standard deviations for the engineer's estimates and the prior worths, he can calculate the weights for the prior and the engineer's estimates (as above). Use E for the engineer's evaluation of the present worth of the benefits, and P for the prior estimate. Let the weights for the engineer's evaluation be W_e (this is the worth from textbook type calculations) and the weight for the prior be W_p (judgment). The posterior estimate is then W_pP+W_eE. Our decision rule is the usual textbook one, to accept the project if this estimate exceeds the cost, C. The cost here is presumed to be known with certainty and to be either funds expended at the start of the first period, or to be the present worth of the costs if they are expended at different times. The textbook rule becomes, accept if, $W_pP+W_eE > C$.
This implies, accept if $W_eE > C\text{-}W_pP$, which also implies, accept if $E > (1/W_e)(C\text{-}W_pP)$. Finally, dividing both sides by C gives:

$$E/C > (1/W_e)\left(\frac{C - W_P P}{C}\right)$$

This is a very important result, since E/C is merely the benefit-cost ratio. (E was the engineer's estimate for the present worth of the benefits). The conventional textbook rule is to accept a project if the calculated benefit-cost ratio exceeds one. This analysis shows that the textbook rule is wrong. Instead, the above calculation provides a simple formula for the right minimum acceptable estimated benefit-cost ratio.

As a plausibility check, if the weight given to the prior is zero, the weight given to the engineer's evaluation is 1, this equation simplifies down to accept if $E/C > 1$. This is the textbook rule, accept if the benefit-cost ratio exceeds one.

Just as an example, the project's cost may be known to be $100,000, and the prior for the present worth of benefits (revenues minus operating costs) may be $50,000, for an index of .5. The quality of the data may be such that the prior knowledge receives a weight of .5, and evaluations of the firm's engineers receives a weight of .5. Then the minimum acceptable benefit-cost ratio is (1/.5)(1-.5*.5) or 1.5.

Generally, the lower the prior estimate of the benefits, and the lower the weight given to the engineer's evaluation, the higher the critical benefit-cost ratio should be.

OTHER IMPLICATIONS

It has been shown that for one special case (normal prior, normally distributed engineer's evaluation) the resulting distribution is normal, with the mean being a weighted average of the means for the prior and the engineer's (sample) information. The weights depend on the relative standard deviations of the two distributions. These weights can be interpreted as measures of the amount of information in the prior and in the engineer's information.

Of course, it is not always true that normal distributions are correct descriptions of the relevant distributions. However, in some other cases, the optimum posterior mean estimate can be approximated as a weighted average of the means for the prior distribution and the engineer's estimate. Since the above results regarding the benefit-cost ratio are expressed in terms of the weights for the prior estimates and for the engineer's estimates, the above results can be applied in these cases.

The most common reason for having a prior belief that good projects are scarcer than poor projects is that competitors could do the same projects, and they will have either taken these projects, or will have bid up unique resources required by them. However, this does not apply for all projects. Projects that involve maintenance of machines, modernization of equipment, etc. can frequently be conducted by only one firm, the firm that owns the facility. While it is probable that there are still more such bad ideas than good ones, the minimum acceptable benefit-cost ratio for such projects should be less than for projects that competitors have access to.

For profit-making firms, business finance texts (Brigham and Gapenski, 1985, Gitman, 1994, Seitz & Ellison, 1995) sometimes use the profitability index, which is the ratio of the present value (present worth) of future operating profits to the project's cost. The rule is to accept the project if the profitability index exceeds one. However, the profitability index is mathematically the same as the benefit-cost ratio, with benefits defined as operating profits. Thus, the logic of the above argument implies that the optimal value for the minimum acceptable profitability index will generally differ from one. For the common case where firms are competing for good projects, the profitability index required for acceptance should exceed one.

CONCLUSION

There is prior knowledge that poor projects are more common than good projects. This is a prediction of economic theory that arises from competitors taking the best projects and bidding up the costs of resources required for projects.

Bayes' Theorem can be used to optimally combine prior knowledge with the engineer's estimates. Frequently the result is a weighted average of the prior present worth for benefits and the engineer's estimates of present worth. When this happens, a simple rule for the minimum acceptable benefit-cost ratio can be derived.

This simple rule is to accept a project if the estimated benefit-cost ratio exceeds a critical value, which is, $(1/W_e)(1-W_p P/C)$. For projects that competitors have access to, this will exceed the traditional cut off ratio of unity. While similar rules could be derived for other capital budgeting criteria, benefit-cost ratios are probably the simplest to work with for practical problems.

REFERENCES

1. BOROKHOVICH, KENNETH A., ROBERT J. BRICKER, TERRY L. ZIVNEY, and SRINIVASAN SUNDARAM, "Financial Management (1972-1994): A Retrospective," *Financial Management*, Vol. 24, No. 2, 1995 (Summer), pp. 42-53.
2. BRIGHAM, EUGENE F. and LOUIS C. GAPENSKI, *Intermediate Financial Management*, The Dryden Press, Chicago, 1985.
3. DYCKMAN, T. R., S. SMIDT, and A. K. MCADAMS, "Management Decision Making Under Uncertainty," MacMillan, London, 1969.
4. ESCHENBACH, TED G., *Engineering Economy*, Irwin, Chicago, 1995.
5. GITMAN, LAWRENCE J., *Principles of Managerial Finance*, Harper Collins, New York, 1994.
6. MILLER, EDWARD M., *Study of Energy Fuel Resources*, Abt Associates, Cambridge, Mass., 1969.
7. MILLER, EDWARD M, "Uncertainty Induced Bias in Capital Budgeting," *Financial Management*, Vol. VII, No. 3, 1978 (Autumn) pp. 12-18.
8. MILLER, EDWARD M., "Capital Budgeting Errors Seldom Cancel," *Financial Practice and Education*, 2000 Fall, pp. 128-135.
9. NEWMAN, DONALD G. and JEROME P. LAVELLE, *Engineering Economic Analysis*, Engineering Press Inc. 1998.
10. PARK, CHAN S., *Contemporary Engineering Economics*, Addison Wesley Longman, Inc. 1996.
11. PARK, CHAN S., GUNTER P. SHARP-BETTE, *Advanced Engineering Economics*, Wiley, John & Sons, Inc. New York, 1990.
12. RIGGS, JAMES L, DAVID D. BEDWORTH, and S.U. RANDHAWA, *Engineering Economics*, McGraw-Hill, New York, 1996.
13. SEITZ, NEIL and MITCH ELLISON, *Capital Budgeting and Long-Term Financing Decisions*, Dryden, New York, 1995.
14. STEINER, HENRY MALCOLM, 1996. *Engineering Economic Principles*, 1996. New York: McGraw Hill.

BIOGRAPHICAL SKETCH

EDWARD MILLER is research professor of economics and finance at the University of New Orleans. Professor Miller is a research professor of Economics and Finance at the University of New Orleans. His Ph.D is in economics from MIT, where his thesis was on the benefits and costs of water resource projects. Practical experience includes stints in the US Department of Transportation, The Department of Treasury, The White House, and The Office of Management and Budget. His research interests are in investment and capital budgeting.